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ON APPEL'S EQUATIONS IN NONLINEAR QUASI-ACCELERATIONS AND QUASI-VELOCITIES*

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Quasi-velocities and quasi-accelerations linked by nonlinear relations to generalized velocities and derivatives of quasi-velocities respectively, are proposed for defining motions of a mechanical system. It is shown that the equations of motion in terms of nonlinear quasi-accelerations and quasi-velocities can be of the form of Appel's equations. Equations of motion of a natural trihedron of a material point trajectory is presented as an example.

1. The motion of a mechanical system with n degrees of freedom, and ideal and holonomic constraints is considered. The configuration of system is defined in independent generalized coordinates $q = \{q_1, \ldots, q_n\}$.

In conformity with /1–4/ and others, we introduce quasi-velocities $\omega = \{\omega_1, \ldots, \omega_n\}$ to enable us to represent generalized velocities in the form of functions of quasi-velocities, generalized coordinates, and time. We also introduce quasi-accelerations $\varepsilon = \{\varepsilon_1, \ldots, \varepsilon_n\}$ linked by nonlinear relations with derivatives of quasi-velocities, and also with quasi-velocities, generalized coordinates, and time, using formulas

$$q^{\cdot} = f(\omega, q, t), \quad \omega^{\cdot} = \varphi(\varepsilon, \omega, q, t) \tag{1.1}$$

The Jacobians $|\partial f/\partial\omega|$ and $|\partial\varphi/\partial\varepsilon|$ are assumed nonzero.

Note that usually quasi-accelerations are simply called derivatives of quasi-velocities. The introduction of quasi-accelerations by the second of relation (1.1) offers new possibilities in the composition of equations of motion.

The Gauss principle is used for deriving the equations of motion

$$\sum_{\nu=1}^{N} (m_\nu \mathbf{r}_\nu^{\cdot\cdot} - \mathbf{F}_\nu)\cdot\delta\mathbf{r}_\nu^{\cdot\cdot} = 0 \tag{1.2}$$

We have

$$\mathbf{r}_\nu = \mathbf{r}_\nu(q, t), \quad \mathbf{r}_\nu^{\cdot} = \frac{\partial\mathbf{r}_\nu}{\partial q} q^{\cdot} + \frac{\partial\mathbf{r}_\nu}{\partial t} = \frac{\partial\mathbf{r}_\nu}{\partial q} f + \frac{\partial\mathbf{r}_\nu}{\partial t}$$

$$\mathbf{r}_\nu^{\cdot\cdot} = \frac{\partial\mathbf{r}_\nu}{\partial q} q^{\cdot\cdot} + \ldots = \frac{\partial\mathbf{r}_\nu}{\partial q}\frac{\partial f}{\partial\omega}\omega^{\cdot} + \ldots = \frac{\partial\mathbf{r}_\nu}{\partial q}\frac{\partial f}{\partial\omega}\varphi + \ldots$$

where terms not containing quasi-accelerations have been omitted.

Note that

$$\frac{\partial\mathbf{r}_\nu^{\cdot\cdot}}{\partial\varepsilon} = \frac{\partial\mathbf{r}_\nu}{\partial q}\frac{\partial f}{\partial\omega}\frac{\partial\varphi}{\partial\varepsilon}$$

Virtual accelerations are obtained using the virtual quasi-accelerations

$$\delta\mathbf{r}_\nu^{\cdot\cdot} = \frac{\partial\mathbf{r}_\nu}{\partial q}\delta q^{\cdot\cdot} = \frac{\partial\mathbf{r}_\nu}{\partial q}\frac{\partial f}{\partial\omega}\delta\omega^{\cdot} = \frac{\partial\mathbf{r}_\nu}{\partial q}\frac{\partial f}{\partial\omega}\frac{\partial\varphi}{\partial\varepsilon}\delta\varepsilon = \frac{\partial\mathbf{r}_\nu^{\cdot\cdot}}{\partial\varepsilon}\delta\varepsilon$$

Transformation of Eq.(1.2) yields

$$\sum_{\nu=1}^{N} (m_\nu \mathbf{r}_\nu^{\cdot\cdot} - \mathbf{F}_\nu)\cdot\frac{\partial\mathbf{r}_\nu^{\cdot\cdot}}{\partial\varepsilon}\delta\varepsilon = \left(\frac{\partial S}{\partial\varepsilon} - Q_\varepsilon\right)\delta\varepsilon = 0 \tag{1.3}$$

$$S = \frac{1}{2}\sum_{\nu=1}^{N} m_\nu \mathbf{r}_\nu^{\cdot\cdot}\cdot\mathbf{r}_\nu^{\cdot\cdot}, \quad Q_\varepsilon = \sum_{\nu=1}^{N}\mathbf{F}_\nu\cdot\frac{\partial\mathbf{r}_\nu^{\cdot\cdot}}{\partial\varepsilon}$$

where S is the energy of accelerations and Q_ε are the generalized forces that correspond to quasi-accelerations ε.

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Noting that $\delta\varepsilon$ is an arbitrary vector, we have the equations of motion in Appel's form in terms of nonlinear quasi-accelerations and quasi-velocities

$$\partial S/\partial\varepsilon = Q_\varepsilon \tag{1.4}$$

2. Example. Motion of the natural trihedron of a material point trajectory /5/.

Position of the point is determined in Cartesian coordinates x, y, z. The quasi-velocities $\omega = \{V, \lambda, \mu\}$ and quasi-accelerations $\varepsilon = \{w_\tau, \Omega_b, \nu\}$ are defined by the equations

$$x^\cdot = V\cos\mu\cos\lambda,\quad y^\cdot = V\sin\mu,\quad z^\cdot = -V\cos\mu\sin\lambda \tag{2.1}$$

$$V^{\cdot} = w_\tau,\quad -\lambda^{\cdot}\cos\mu\sin\nu + \mu^{\cdot}\cos\nu = \Omega_b \tag{2.2}$$

$$\lambda^{\cdot}\cos\mu\cos\nu + \mu^{\cdot}\sin\nu = 0$$

where V is the projection of the point velocity vector on the tangent to the trajectory, λ is the heading angle, μ is the angle of climb, w_τ is the tangential acceleration, Ω_b is the projection of angular velocity vector of the natural trihedron of the trajectory on the binormal, and ν is one of the angles that define the natural trihedron orientation.

The last of Eqs.(2.2) represents the condition of zero projection of the angular velocity vector of the natural trihedron on the normal to the trajectory. Solving Eqs.(2.2) for derivatives of quasi-velocities we obtain

$$V^{\cdot} = w_\tau,\quad \lambda^{\cdot} = -\Omega_b\sin\nu/\cos\mu,\quad \mu^{\cdot} = \Omega_b\cos\nu \tag{2.3}$$

The energy of accelerations and generalized forces

$$S = m\,(x^{\cdot\cdot 2} + y^{\cdot\cdot 2} + z^{\cdot\cdot 2})/2 = m\,(w_\tau^2 + V^2\Omega_b^2)/2$$

$$Q_{w_\tau} = F_\tau,\quad Q_{\Omega_b} = VF_n,\quad Q_\nu = V\Omega_b F_b$$

where F_τ, F_n, F_b are projections of the force applied to the material point on the tangent, the normal, and the binormal.

Appel's equations in nonlinear quasi-accelerations and quasi-velocities are of the form

$$mw_\tau = F_\tau,\quad mV\Omega_b = F_n,\quad 0 = F_b \tag{2.4}$$

which are the same as the well-known equations of motion of the natural trihedron /5/. They must be considered together with formulas (2.1) and (2.3).

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